The Rules For The Final

1) If your grade is less than a C (72%), you **must** take the final!

2) If you have an A (92%) either before or after completing the last homework, you are exempt from the final

3) If your grade is between 72% and 92%, you can either

a) Opt out of the final right now before completing the last homework

b) Opt out of the final after completing the last homework

c) Take the final.

In cases a) and b), you must send me an e-mail declaring your intention to skip the final (and skip the last homework, if applicable).

4) If you take the final, your course grade will be determined via taking the maximum of

a) Your score on the final

b) Your overall course grade, computed with the final.

The final can both help and hurt your grade!

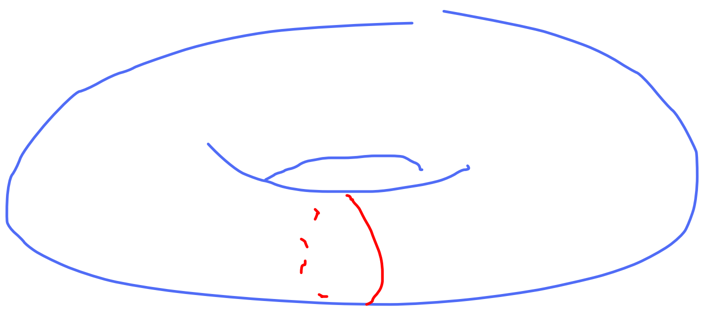
Easy way to check whether a vector field on $\mathbb{R}^3$ is conservative

If $\vec{F} = \langle P, Q, R \rangle$ and the first order partial derivatives of P, Q, and R are continuous, then if $\operatorname{curl}(\vec{F}) = \langle 0, 0, 0 \rangle$, then $\vec{F}$ is conservative

Surfaces

We aren't sophisticated enough to define this rigorously, but here are some pictures

Hollow cap
(half a sphere)



Hollow doughnut

A parametric surface

is the image of all points in $\mathbb{R}^3$ that are outputs of

$$\boxed{\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle}$$

where $x, y, z : \mathbb{R}^2 \to \mathbb{R}$

and (u,v) are in some plane region R (usually bounded)

Example 1: Just like the graph of any function $y=f(x)$ is trivially parameterized by $\vec{r}(t) = \langle t, f(t) \rangle$,

any function $z = g(x,y)$ is parameterized by

$$\vec{r}(u,v) = \langle u, v, g(u,v) \rangle .$$

Example 2: The sphere of radius one and center $(0,0,0)$ is not the graph of a function $z = g(x,y)$, but we can parameterize it using spherical coordinates:

$$\vec{r}(u,v) = \langle \cos(u)\sin(v), \sin(u)\sin(v), \cos(v) \rangle$$

$$(\rho = 1)$$

Integration Over a Surface

Let

$r(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

for (u,v) in a plane region R

define a surface in $\mathbb{R}^3$.

Set

$$\vec{r}_u = \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle \text{ and}$$

$$\vec{r}_v = \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right\rangle.$$

Suppose

$$\vec{r}_u \times \vec{r}_v \neq \langle 0, 0, 0 \rangle.$$

Then set

$$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{\| \vec{r}_u \times \vec{r}_v \|}.$$

We then define the

Surface integral of the

vector field

$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$

over the surface S

parameterized by

$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

as

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$= \iint_S \vec{F} \cdot \vec{n} \, dS$$

$$= \iint_D \left(\vec{F}(\vec{r}(u,v)) \cdot \vec{n} \; \|\vec{r}_u \times \vec{r}_v\| \, dA \right.$$

$$= \iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v) \, dA$$

This is the flux of $\vec{F}$ over S.